On lepton pair production by muons

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Abstract
The paper considers the lepton pair production by muons using the Geant4 toolkit for the Monte Carlo simulation of the high energy muon transport. The new Geant4 process is described for the muon pair production. It is shown, that during the high-energy muon pair production, the cross section is about 10^{-5} of the electron-positron pair production cross section, but this process may provide a background to rare processes studied in LHC and FCC experiments.

Keywords Simulation · Geant4 · Muons · Leptons

Introduction

In particle physics, the Geant4 [1–3] is the main engine for a Monte Carlo simulation of the particle transport. It is used to prepare and analyze all Large Hadron Collider (LHC) experiments at the CERN. The registration of muons is an essential component of analyses of reaction channels in proton-proton collisions. The muon transport in the Geant4 is simulated with respect to the main muon interactions [4], which include ionization, bremsstrahlung, electron-positron pair production and muon nuclear process. In this work, we describe the extra muon pair production by muons added in the Geant4 toolkit. This process has a lower cross section than the main muon processes but may provide a specific background for new physics search at the LHC and the Future Circular Collider (FCC).

Kelner [5] in his initial deep theoretical study assumed a point-like nuclear charge. The muon pair production was observed in the BARS liquid calorimeter [6] and in cosmic run of the ALEPH experiment [7]. After the first observation, Kelner and co-authors proposed an extended approach to the muon pair production [8] considering the finite nuclear size and both low and high screening effects.

The aim of this work is to present the code developed for the Geant4 toolkit and the results obtained. Preliminary reports were done in works [9, 10] in the frame of the CERN summer school program.

Theory of lepton pair production

In the Geant4 algorithm of the muon pair production model, differential cross sections $\sigma(E, v, u)$ are implemented by the approximated relativistic Eq. 15 from [8]:

$$d^2\sigma(\mathrm{E, v, u}) = \frac{2}{3\pi}\left(Z\alpha r_\mu\right)^2 \frac{1-v}{v}\Phi(v, u)ln(X)dvdu,$$

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where E is the primary muon energy, Z is the atomic number of the target nucleus, α is the fine structure constant (the Planck constant and speed of light in vacuum are equal to unit), r_μ is the muon classical radius, $\Phi(v, u)$ is the kinematic density function (see Eq. 18 in [8]), X is the function, which considers the nuclear size and screening (see Eq. 20 in [8]). The reaction kinematics is defined in the following way:

$$v = \frac{E - E'}{E} = \frac{E_+ + E_-}{E}, u = \frac{E_+ - E_-}{E_+ + E_-},$$

where $E' = E(1 - v)$ is the final muon energy, E_+, E_- are energies of produced leptons. Kinematic variables v and u are limited by inequalities

$$\frac{2\mu}{E} \le v \le 1 - \frac{\mu}{E}, |u| \le u_{max} = 1 - \frac{2\mu}{vE},$$

where μ is the muon mass, v and u are unitless kinematic variables. This differential cross section is convenient for making the Monte Carlo algorithm, because it provides a two-dimensional distribution in the limited areas of sampled variables. The Geant4 class *G4MuonToMuonPairProductionModel* implements sampling of the muon pair production. The class inherits of the based class *G4MuPairProductionModel,* and the same approach is implemented: at initialization of the Geant4 sampling, tables are prepared for 5 values of Z and several points in the energy of primary muons logarithmically distributed over the interval from 850 MeV to the maximum energy (100 TeV default and respectively 22 energy points). The numerical integration by the 8-point Simpson method is performed over a differential cross section to compute the process cross section. The approximate method proposed in [4] is implemented for angular sampling of the final lepton pair.

This new process of the muon pair production, is included in top of any prepackaged Geant4 Physics List using class *G4MuonToMuonPairProduction,* which is optional and may be enabled *via* the Geant4 UI commands*/physics_lists/em/MuonToMuons true*, which should be issued before start of the Geant4 physics initialization.

Results and discussion

The example of cross sections as a function of the muon energy for the iron target is shown in Fig. 1. The ratio between the electron and muon pair production is almost constant (~10^{-5}) at high energies of the projectile muon. The choice of the low-energy limit connects with the limited accuracy of the approximation used in these formulas. At this threshold, the cross section is small, so this limit is acceptable for the simulation of high energy experiments.

The spectrum of the energy transfer to the lepton pair in iron is shown in Fig. 2. The energy transfer to the electron-positron pair is significantly lower than to the muon pair. The muon pair production is thus a not common process, but when it happens, the high-energy muon splits into 3 energetic muons, which may provide a specific signature in the muon system of the experiment and generate a background for the search of new physics. According to Fig. 2, the distribution tail toward the low energy transfer to the muon pair, is low.

The lepton pair production is a background process for the MUonE muon scattering experiment at 160 GeV muon beam at CERN [11]. The experiment is being prepared for precise measurements of the muon scattering off the target atomic electrons. The experiment aims at a precise measurement of α_μ value, i.e., hadronic vacuum polarization to the muon magnetic moment anomaly. This measurement is an alternative to precise measurement of the hadron production at electron-positron colliders. Abbiendi et al. [11], present a detailed analysis of Monte Carlo generators Geant4, MESMER [12] and McMule [13]. The last two specialized generators consider all diagrams involved in the reaction of the lepton pair production, while the Geant4 model is based on approximate

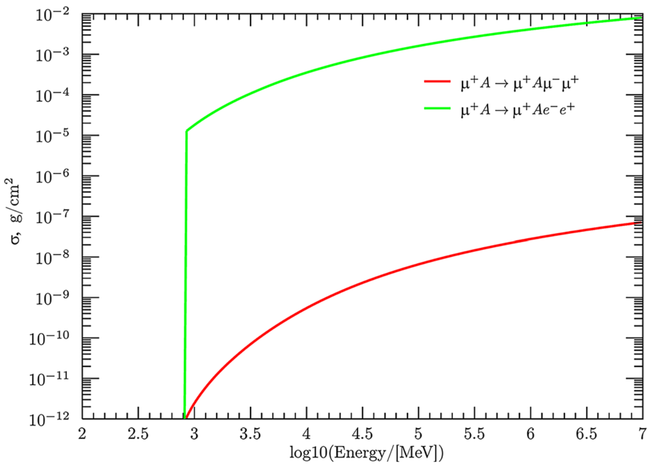

Fig. 1 Geant4 cross sections of lepton pair production by muons in iron as a function of muon energy. *Red* and *green* curves indicate muon and electron-positron pairs, respectively. Energy threshold: 850 MeV

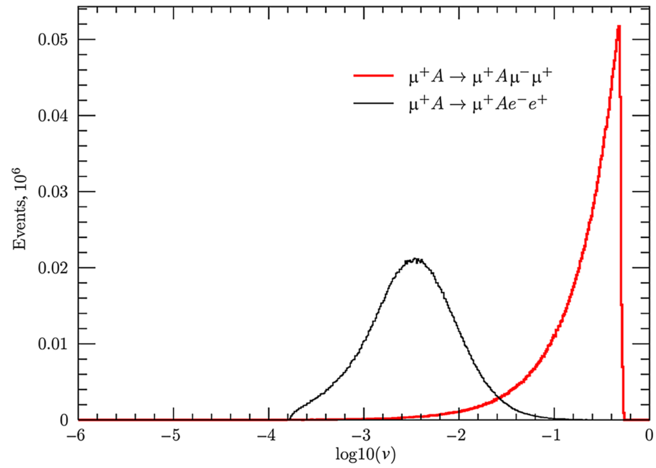

Fig. 2 Geant4 energy transfer spectra of lepton pair production by muons in iron. *Red* and *black* curves indicate muon and electron-positron pairs, respectively, ν is the relative energy primary muon-to-pair transfer $\nu = \frac{(E_+ + E_-)}{E}$

formulas proposed in [8]. The difference identified for cross sections between Geant4 and MESMER does not exceed 4%.

Conclusions

Geant4 models for the electron-positron pair production were introduced before the LHC start and provided good accuracy for LHC experiments. The paper reported on the code status for the new Geant4 process of the muon pair production, whose cross section was significantly lower, but should be considered in ATLAS and CMS experiments for different analyses. This process should be also considered in preparing the physics program for the FCC. It was already checked and used in the CERN MUonE experiment simulation.

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Data availability The authors confirm that all data generated or analyzed during this study are included in this published article.

Conflict of interest The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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